ICMI23 Special Needs Discussion Panel

Research and Instruction in Whole Number Arithmetic (WNA)

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Research Focus

- Are MLD linked to a lack of underlying awareness of mathematical patterns and relationships?
- Is mathematical development linked primarily to spatial ability rather than development of number?
- How is spatial reasoning/ability linked to awareness of mathematical pattern and structure?
- (role of transformational ability and co linearity)

Transdisciplinary approach

- Conceptual connectivity within and between domains of knowledge (or disciplines).
- Spatial reasoning as central to mathematical development (Spatial Reasoning Research Group; Davis et al. 2015)
- Interaction with ennvironment vs cognitive neuropsychological development? (Woolcott & Mulligan, PME 2015; ICMI 23)
- Number and spatial relations under investigation

Broad Research Questions

- Is ‘pattern and structure’ a general underlying construct that is generic to all mathematics learning?
- Can it be described and measured?
- What is the role of spatial ability and perception in mathematics learning?
- Why do low achievers fail to recognise pattern and structure ?
- Can a teaching and learning program promote pattern and structure?

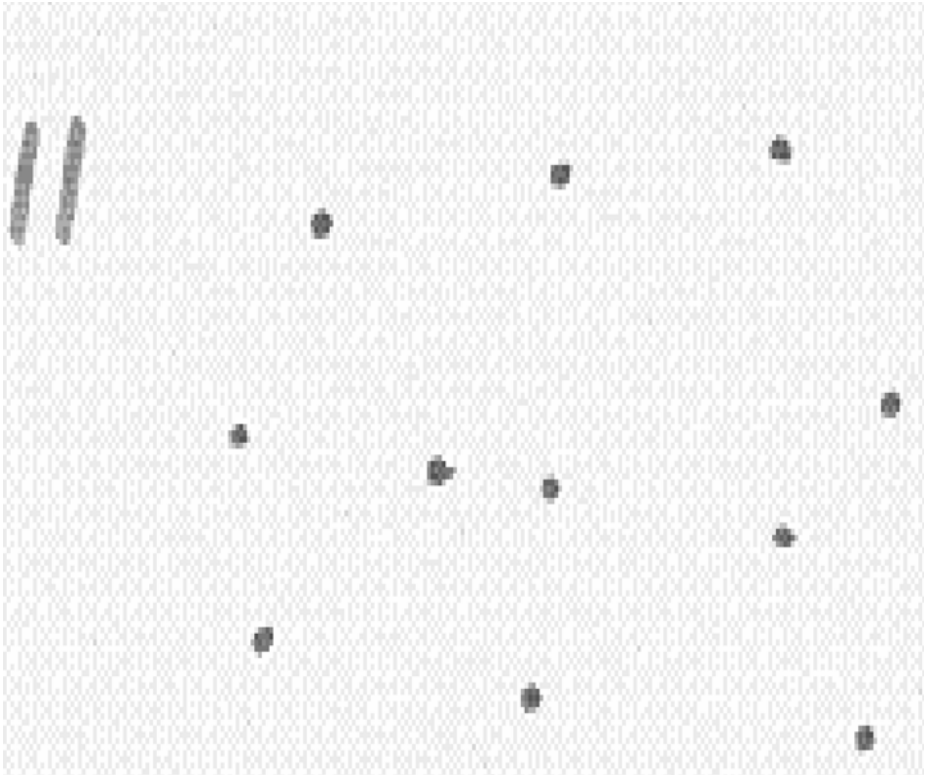
Theoretical Influences

- Neuroscientific influences (Dehaene; Butterworth; Reeves; Geary)
- Cognitive psychology: Structural development of representations and problem-solving (Goldin, 1994; Thomas Mulligan, Goldin, 2002)
- Imagery and processing (Gray, Pitta & Tall, 1996)
- Spatial visualisation: pattern imagery (Presmeg, 1998)
- Early algebraic thinking (Blanton & Kaput, 2004; Brizuela & Earnest, 2004; Carraher et al. 2007; Papic & Mulligan, 2011; Schliemann et al., 2005; Warren, 2004; Warren & Cooper,2008)
- Spatial structuring (Battista, 2003; van Nes, 2007)

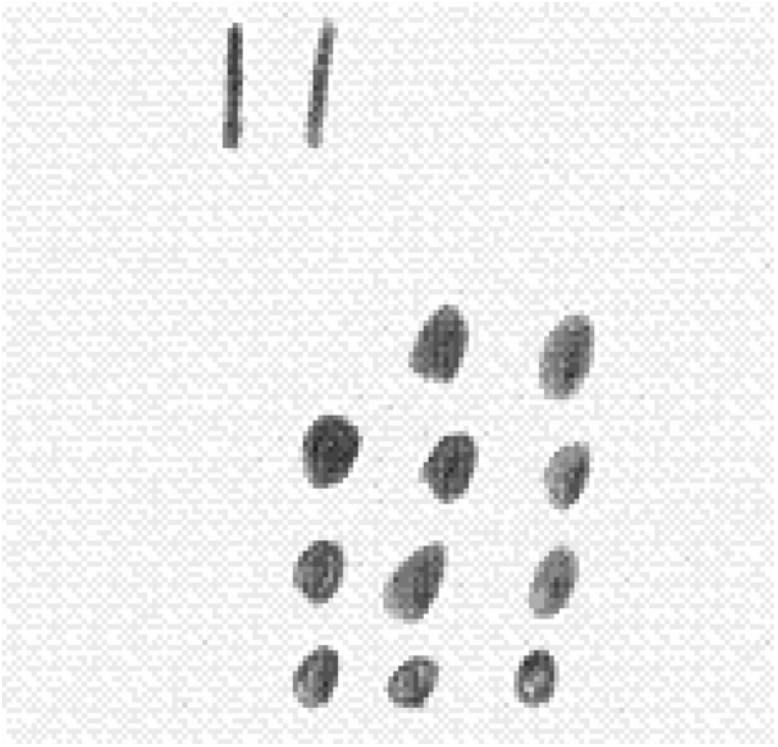
What is Awareness of Mathematical Pattern and Structure (AMPS)?

- A **pattern** is any predictable regularity in our environment
- **Structure** is the way a pattern is organised and develops relationships
- Types of mathematical patterns:
  - Repeating Patterns: ‘unit of repeat’ ABC ABC ABC
  - Growing Patterns: increase or decrease systematically e.g. triangular number pattern 1, 3, 6, 10, 15
  - Spatial Patterns: 2D and 3D designs, tessellations, transformations
  - Functions: relations between variables

Early underlying awareness of pattern and structure: Why? How?



“I see 11 all over as I count”



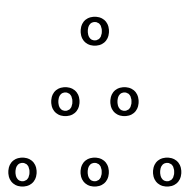
“I see 11 as a square of 3s and 2 more”

Awareness of Mathematical Pattern and Structure (AMPS)

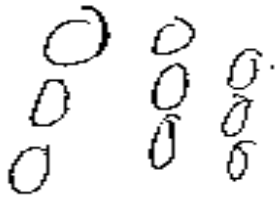
- Young children's use of mathematical pattern and structure tends to be similar across mathematical concepts
- The general underlying construct is called Awareness of Mathematical Pattern and Structure (AMPS)
- AMPS has two components: recognition of common structures and a tendency to look for patterns
- Early school mathematics achievement is associated with children's level of AMPS

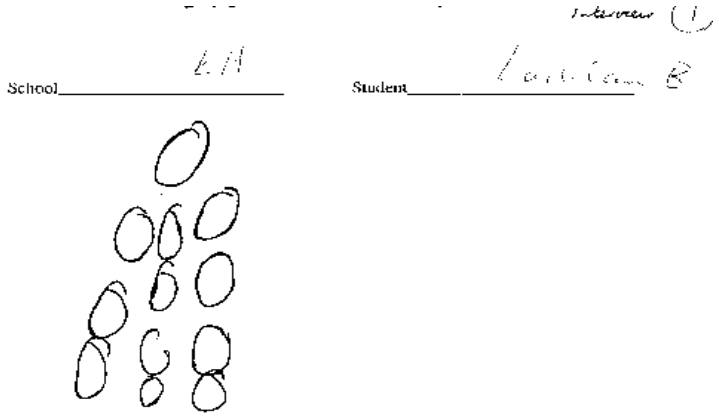
Triangular Pattern Task

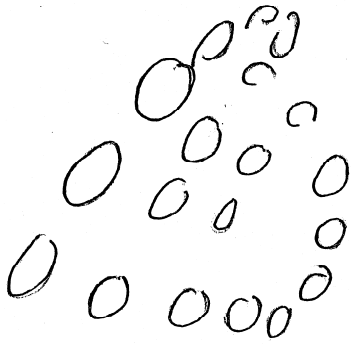
(Flash card with pattern) Look carefully. Cover. Draw exactly what you saw. Describe it.



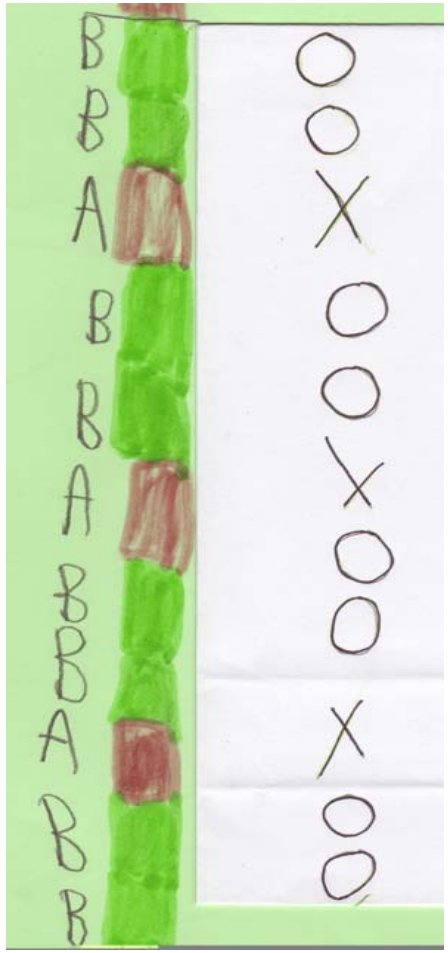
Triangular Pattern: Responses Over Time





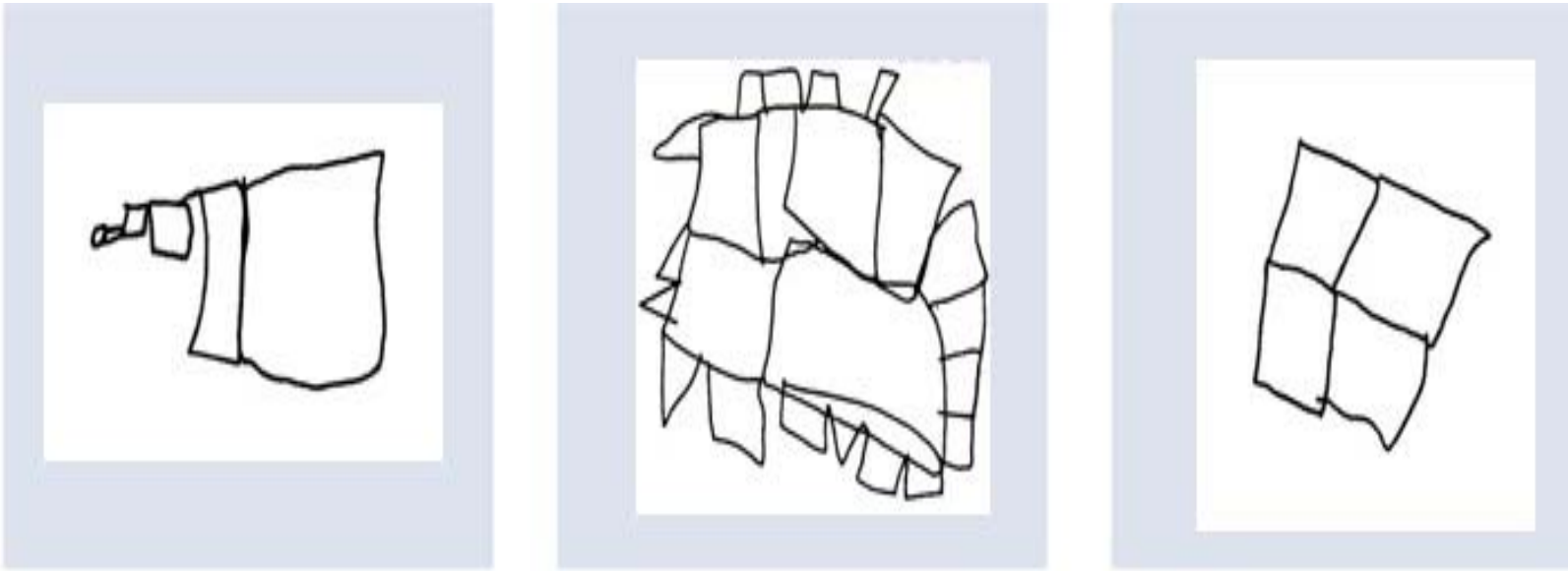


Repetition as Emergent Generalisation: Same Structure/Different Symbols



Tracking Individual Structural Development

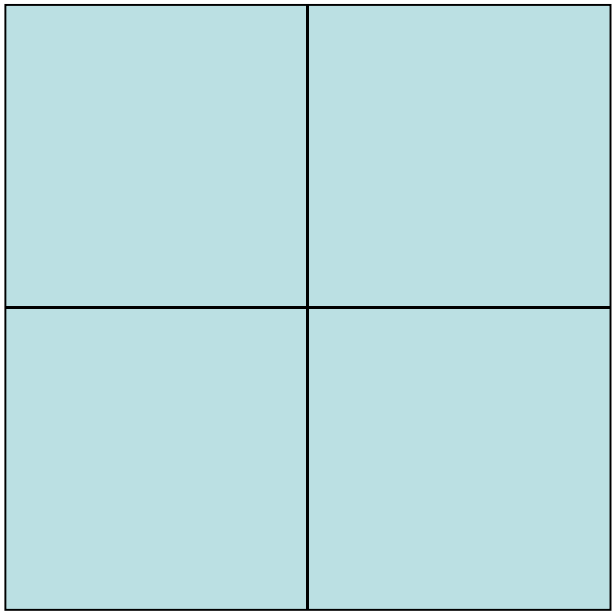
Constructing and Recording Grids



“I made them the same…the squares have the same on each side. It doesn’t matter if they are big or small, they got the same sides. You have to put only the squares that you need. They have to be same size… I know they have to match if they are on top and on top you know like we made with the puzzles… I made a four with two and two…you can go both ways ”.

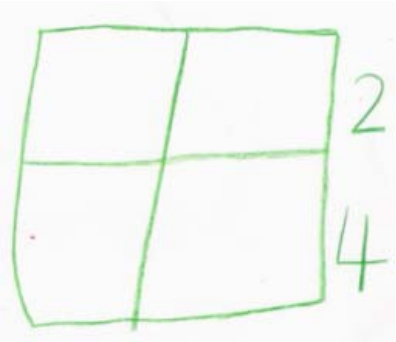
Focus on spatial structuring, doubling using collinearity (2 dimensionality), recognition of part/whole relationship, congruence and similarity, ‘unit of repeat’ / composite units

Using Spatial Structuring: Grid Structure (collinearity)



A “window”

Using Spatial Structure to Develop Number

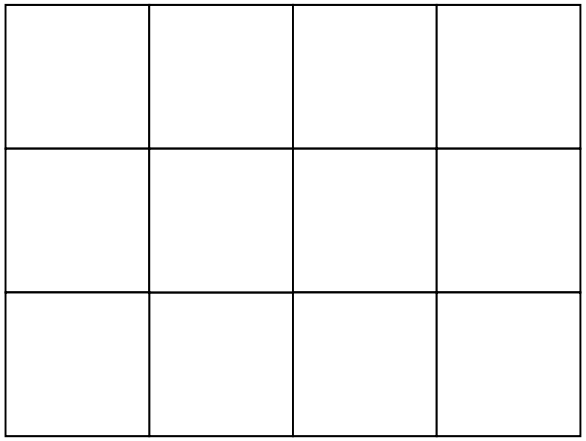


Heela

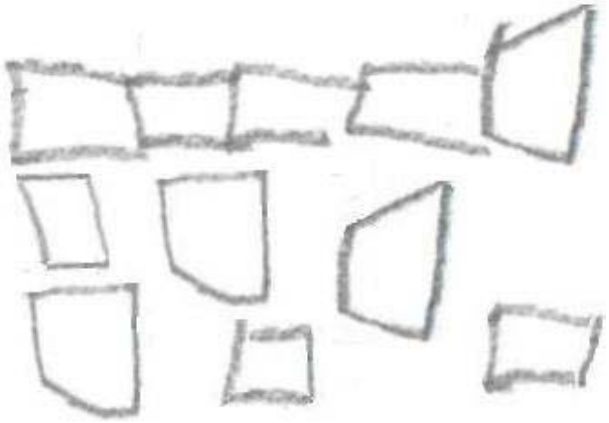
Lateh

Typical student responses

Using Grid Structure to Model Area



Stimulus



Response

The Pattern & Structure Mathematical Awareness Program (PASMAP)

- Scaffolded learning sequences highlight pattern and structure
- In pattern-eliciting tasks, students make representations, often from memory
- Teacher draws attention to mathematical features – emphasis on "sameness" and "difference"
- Tasks gradually become more complex and link to other concepts
- Explanation and justification encourage simple symbolisation and emergent generalisation

PASMAP Approach

- Highlight pattern and structure in scaffolded learning sequences
- Draw attention to mathematical features – “sameness” and “difference”
- Explicit focus on one aspect of structure at a time
- Encourage connectedness between structural features
- Visual memory activities: recording from memory
- Representations constructed by student
- Explain and justify: encourage simple symbolisation and emergent generalisation
- Measurement and spatial structuring as a basis for number concepts: unitising as a common process

Projects' Research Findings

- Students in year long 'Patterns' program were more advanced than 'Regular' program students in mathematics 12 months after the program finished
- Students in the 'Patterns' program were able to generalise and use algebraic thinking
- Awareness of pattern and structure can facilitate learning of, and connections between, key mathematical processes
- Students experiencing difficulty in learning mathematics do not recognise pattern and structure; increasingly confused and 'crowded' mathematical ideas and representations

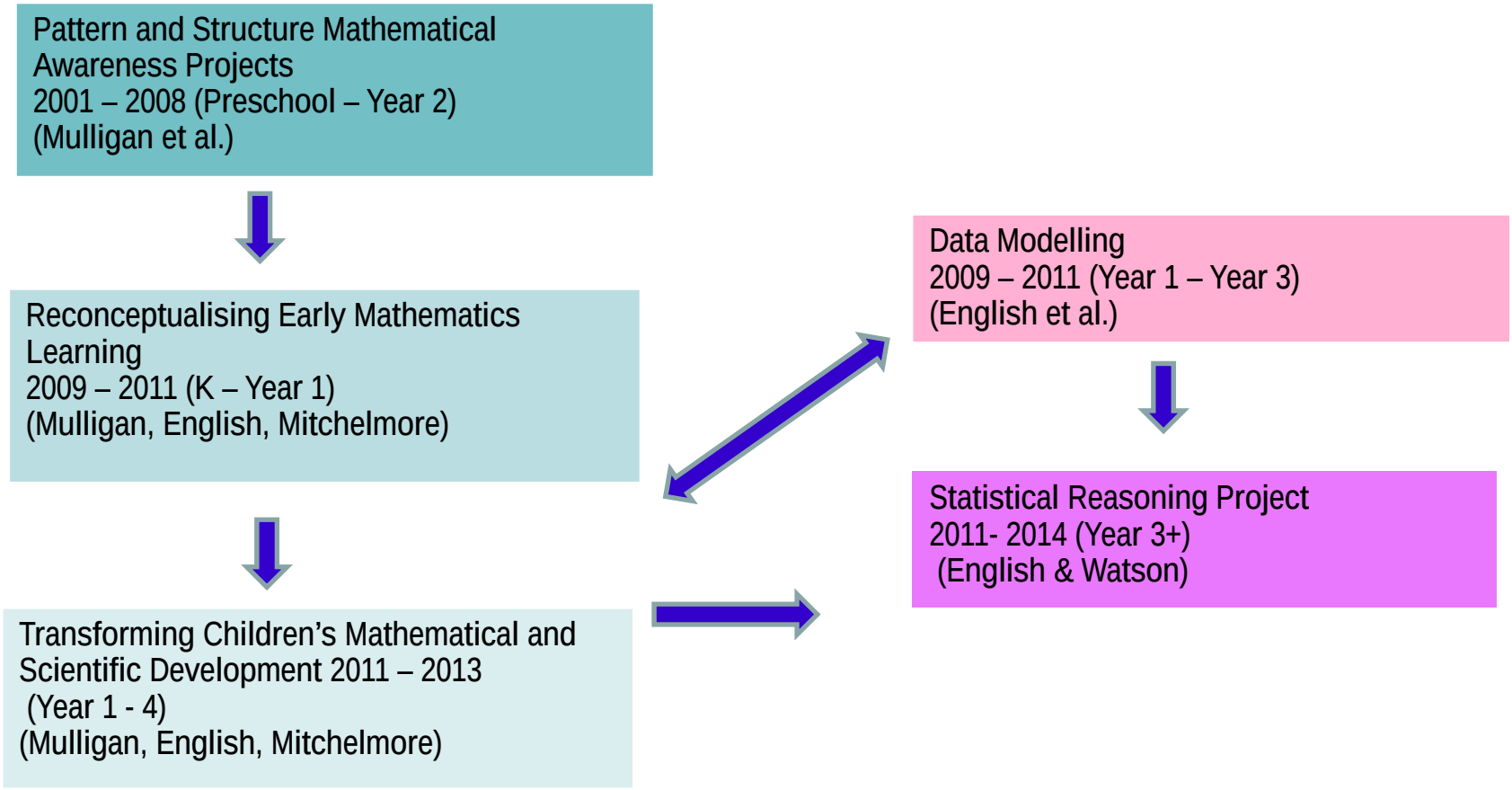
Findings: Classroom (Lesson Episodes)

- PASMAP students demonstrated abstract thinking and reasoning; formed emergent generalisations
- Engaged for long periods of time
- Stark differences between PASMAP and Regular students' process of learning: PASMAP approach focused on similarity and difference, connecting each aspect of the learning sequence
- Highly able PASMAP students developed systems that had common structural features eg pattern of squares, multiples, commutativity and 100 square
- Regular students demonstrated achievement of the expected outcomes in the same ways: no differentiation
- Most PASMAP students demonstrated learning outcomes well beyond Kindergarten curriculum expectations

Conclusions

- There is a wide range of AMPS in the average primary class
- PASMAP-type tasks can help students develop their AMPS
- Central principle is to exploit children's natural tendency to look for patterns and make generalisations
- It is important to provide scaffolding but not structuring
- Drawing from memory is a particularly valuable activity
- Emphasis on finding generalisations can pay dividends later
- Mathematics curriculum needs to promote structural thinking

Related Projects 2002-2014: A structural approach to early mathematical development



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